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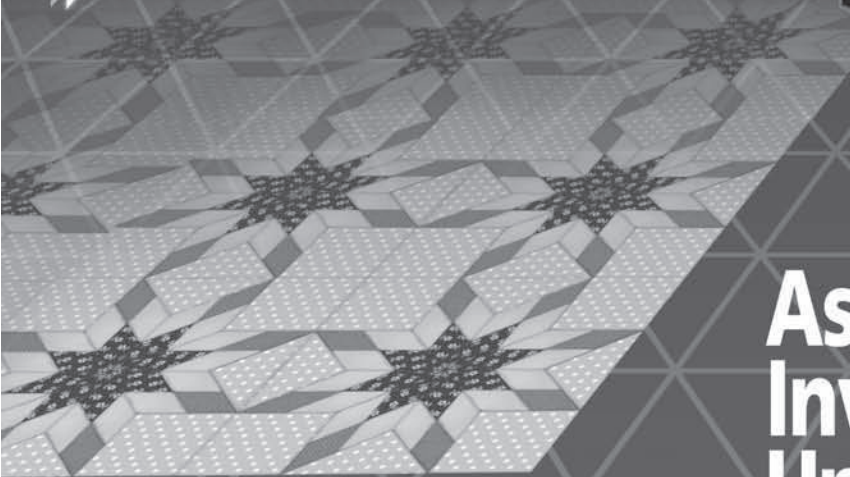
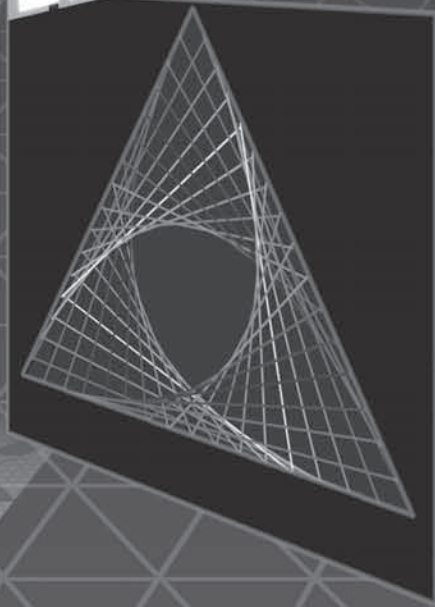
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Grades 5–8



MATH AMAZEMENTS



**Astounding
Investigations
Uncover Math
in Your World**

Pamela Marx

DEDICATION

This book is dedicated to Mark
and to Siv Si, whose teaching energy inspired it.

ACKNOWLEDGMENTS

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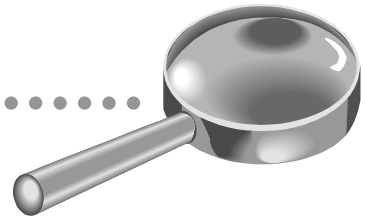
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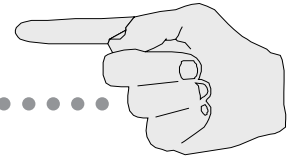


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INTRODUCTION

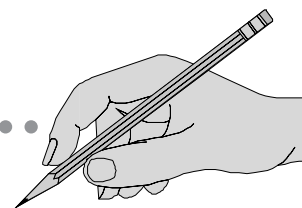


Sometimes we think that math is multiplication and division, addition and subtraction. It is equations and probability and ratios and fractions. As we get older, it is algebra and trigonometry. Math is all of those things, but it is so much more. And most of what makes up math is just plain fun and interesting. This book takes a walk through history, our environment, math thinking, and logic to provide explorations and activities that show how much math there is in the everyday things around us and how much fun it can be to search for the math in things.

The activities in this book offer great ways to stimulate your math thinking. For most people, while some kinds of math thinking and activities are easy, others are difficult. This is true for almost everybody. It simply depends upon how your brain works. Sometimes what is hard or easy depends on when you were born in the history of the world. Nobody would say the ancient Egyptians were not a smart, advanced people, but guess what? They didn't know how to multiply like we do because they didn't really need to and the method wasn't developed yet. Some of the activities in this book will help you see how math thinking has developed through the history of all people. Today, we know a lot of things by third grade that some people in history never knew, and, yet, they built the great pyramids.

Keep all of this in mind as you explore the activities in this book. You might think some of the activities in this book are really easy. Some of them will be easy for you to solve, while others, such as certain kinds of puzzles, may be hard for you but easy for your friend. This is not a book of right and wrong answers. It's a book to give you ways to think about numbers and shapes and surfaces in a new way. Above all, it's a book to have fun with while you are learning about new ideas, relationships, and concepts.

HOW TO USE THIS BOOK



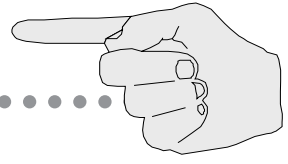
This book includes thirty-five sets of activities, an answer key, a glossary, and a bibliography. The activities provide explorations in areas of math thinking such as geometry, topology, interesting properties of numbers, logic, and probability. The activities will get you thinking about geometry, symmetry, topology, math history, number properties, probability, ratios, puzzles, and games.

Each activity is described on two (and sometimes more) pages. The first page provides some background and introduction to the particular activity. The pages that follow give you ideas and opportunities for exploring the subject. In the “Your Turn” section, you will have a chance to investigate the math subject or idea introduced on the first page. In the “And Another Thing” section, you will find additional ways to explore the subject or ideas for doing different, but related, math investigations.

The book’s additional features are an answer key, glossary, and bibliography. The answer key offers answers for many of the questions and puzzles. Some questions simply challenge your thinking. They may have no right or wrong answers or more than one possible answer. Others may have different answers depending upon the approach you take. A quick answer is not the point; the thinking itself is the point. Those questions in the text preceded by a number in a circle ① are answered in the answer key. The glossary offers a way to check your understanding of a term in an activity. The bibliography includes many books that provide wonderful math ideas and curiosities that students of all ages will enjoy. That’s because math can be just plain fun and interesting when we stop thinking about it as a test to be survived. The Internet is also a great source of entertaining and stimulating math inquiries.

Flip through this book. Stop to read when something interests you. Pull out a piece of graph paper and see what wonders will unfold for you. Above all, enjoy!

A NOTE TO TEACHERS AND PARENTS.....



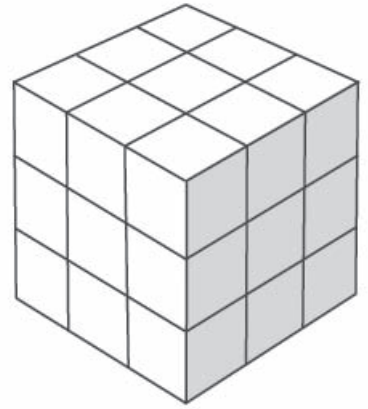
The activities in this book vary greatly in complexity. Some, such as “How the Egyptians Multiplied,” are mathematically quite simple but are interesting to help young people understand the development of math thinking through the ages. Others, such as “The Four-color Map Problem,” are very complicated mathematically, but young people can enjoy exploring the concept or “problem” and some of the initial thinking that goes into solving complicated math problems. Still others, such as the tangram activity, are easy for some and difficult for others, regardless of age. It’s a matter of how our individual brains work. Playing with these puzzles, whether initially easy or difficult, builds abilities and skills.

The activities in this book are written so that they can be enjoyed by young people or presented by an adult who introduces the activity. They are organized so that the activities build in complexity as you progress through the book; however, everyone’s brain works differently and some activities that are easy for one person may be more challenging to another. Each activity is presented on two (or more) pages. A teacher or adult can use the material on the first page of any activity to help present the activity and then provide the young person with the following pages to do an actual hands-on exploration. Alternatively, provide the young person all the pages of an activity to read and investigate. If an activity sparks a particular interest, encourage further exploration. There are many excellent math activity Web sites available, which include particular topics, such as those in this book. You can find these by using keywords in one of the familiar search engines.

Another wonderful use for the activities in this book is as a jumping-off place for group, family, or even community explorations. Teachers might use the activities in this book for special projects, extra-credit assignments and student presentations. Families might use the activities for family fun nights, youth group activities, home schooling, or even parties. Nothing gets people involved in an activity more quickly than exercising their brain cells over something they think should be easy, only to find that it isn’t as easy as they thought.

Remember that the best use of these activities is to provide a stimulating and entertaining way to foster mathematical thinking and to create and develop a pleasure in the process of thinking, creating, and puzzling things out. When it’s all said and done, we learn more that we remember longer when we’re having fun at the same time.

1 FACTORING, GEOMETRIC SHAPES, AND PRIME NUMBERS



The Greeks thought of numbers in groups and gave them geometric names, such as *triangle numbers*, *square numbers*, and *pentagonal numbers*. Positive whole numbers can be analyzed in a geometric way all their own to find out whether or not they are prime and to find out the factors of a number.

What is a *prime number*? It is a whole number greater than 1 that has only itself and 1 as factors. Prime numbers include 1, 2, 3, 5, and 7.

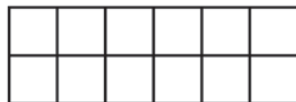
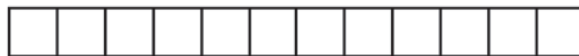
What is a *factor*? Factors are numbers that can be divided into a whole number with no remainder. For example, the factors for the number 8 are 1, 2, 4, and 8: $1 \times 8 = 8$ and $2 \times 4 = 8$. The number 8 has four factors, so it is *not* a prime number.

To find out if a number is prime or not, when you look at it, ask how many kinds of rectangles you can make from it. When you figure this out, you will know whether the number is prime and what its factors are.

For example, look at the numbers 5 and 12. How do you find out if either is a prime number? First, draw a rectangle made of squares for the number 5:



Now look at rectangles for the number 12:



- 1 What are the factors for each number?
- 2 Is either number prime?