GOOD YEAR BOOKS

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MATHEMATICS EXPLORATIONS

Detective-style Activities for the Real World

Aligns to the to the National Council of Teachers of Mathematics Standards and to the Common Core State Standards

David B. Spangler

GOOD YEAR BOOKS

V 2.1

Preface

It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment.
—Karl Friedrich Gauss, German mathematician (1777–1855)

In his landmark text, *The Process of Education,* Jerome Bruner, a founder of cognitive psychology, wrote, "Ideally, interest in the material to be learned is the best stimulus to learn" (1960, p. 12). According to the authors of *Helping Children Learn Mathematics,* "Success in mathematics learning requires being positively disposed toward the subject. Engaging oneself with mathematics requires frequent opportunities to make sense of it, to experience the rewards of making sense of it, and to recognize the benefits of perseverance" (2002, p. 16). The contexts of "mystery and exploration" that permeate the activity lessons in *Mathematics Explorations* are designed to stimulate and promote positive dispositions, beliefs, and attitudes related to the study of mathematics in general and to problem solving in particular. Students who delve into the challenging problems in this sourcebook will likely acquire and refine the disposition of perseverance—and should come away with the belief that exerting effort to learn mathematics is a worthwhile endeavor.

Mathematics Explorations is a sourcebook of engaging mathematics activity lessons that connect to the real world of students in grades 6–9. Both classroom teachers and students in mathematics methods courses should also benefit from the engaging, ready-to-use activity lessons and the professional development that is provided with the Teacher's Notes.

About This Book

Each activity lesson calls for students "to put on a detective hat" and search for patterns to discover important mathematical concepts and formulas, break a code, solve a mystery, conduct detective-type investigations, uncover and correct errors and blunders, analyze why a "trick" works, use clues to solve problems, and more. The activity lessons are designed to arouse student curiosity—resulting in their use of both inductive and deductive reasoning, critical thinking, graphical analysis, and other analytical skills needed for success in school and beyond in the world of work.

Each activity lesson in *Mathematics Explorations* is preceded by one or more pages of Teacher's Notes. The Teacher's Notes include a correlation to the *Common Core State Standards* (Council of Chief State School Officers and the National Governors Association Center for Best Practices, 2010) and to the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). Also included is a list of the mathematics topics covered in the activity lesson and suggestions for student grouping. An extensive Background section provides deep mathematical background, historical notes, possible student responses, related Web sites, and other useful tips for teaching the activity lesson. A variety of mathematical quotations are included to highlight the fact that people from all walks of life

have expressed a strong utility in the study of mathematics. The Teacher's Notes also include Mathematical Humor for each activity lesson. Full Solutions are included for all problems, and an Extension (with solutions) is provided for each activity lesson.

Although students may work through the activities independently, most are enhanced when students work collaboratively in pairs or in small groups. When collaboration occurs, students are encouraged to share and ponder ideas, listen to other points of view and solution processes, and divide the tasks necessary to complete the activity.

Correlation to the Common Core State Standards and to the NCTM Standards

In 2010, the National Governors Board and the Council of Chief State School Officers released the Common Core State Standards for Mathematics. These Standards are organized into two related categories: (1) Standards for Mathematical Content (defining what students should understand and be able to do) and (2) Standards for Mathematical Practice (describing ways in which students should engage with mathematics based on processes and proficiencies). The Standards may be downloaded at http://www.corestandards.org/the-standards. The activity lessons in this book align closely to both categories of standards.

In its *Principles and Standards for School Mathematics,* the National Council of Teachers of Mathematics (NCTM, 2000) addresses five Content Standards and five Process Standards (for acquiring and using the content knowledge) for grades preK–12. These Standards, listed below, are anchored on extensive foundational research on what works in the mathematics classroom.

NCTM Content Standards
Number and Operations
Algebra
Geometry
Measurement
Data Analysis and Probability
NCTM Process Standards
Problem Solving
Reasoning and Proof
Communication
Connections
Representation

Mathematics Explorations addresses key mathematical skills and concepts in each of the five NCTM Content Standards. The book is organized into chapters that provide a focus on each Content Standard. Because most of the activity lessons address multiple Content Standards, students experience how the various mathematics strands are connected. This integration of Content Standards is important for students in their quest to develop mathematical power. According to NCTM's *Principles and Standards for School Mathematics* (p. 64), "The notion that mathematical ideas are connected should permeate the school mathematics experience at all levels."

Mathematics Explorations also fully implements the essence of the NCTM Process Standards. This sourcebook delivers standards-based content via hands-on, discovery learning. This delivery . . .

- promotes the teaching of Problem Solving in context;
- integrates critical thinking and logical reasoning (Reasoning and Proof);
- provides opportunities for students to explain their thinking (Communication);
- makes Connections across mathematical topics to produce a coherent whole (multiple Content Standards are integrated in each

activity lesson) and across disciplines (such as language arts, science, social studies, music, and art); and

- uses modeling and other forms of Representation (such as graphs, number lines, algebra tiles, and simulations).

The Teacher's Notes for each activity lesson provides a correlation to the Common Core State Standards for Mathematics and to the NCTM *Principles and Standards for School Mathematics*.

Because *Mathematics Explorations* builds on important grade-level mathematical content and connections, the book also aligns with the NCTM *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006). The integrated curriculum promoted in *Mathematics Explorations* parallels the integrated nature of the curriculum focal points—which draw upon and integrate multiple content topics and process standards in a single focal point.

Motivating Math Learning with Humor

In the May 2006 issue of *Mathematics Teaching in the Middle School* (p. 419), the Editorial Panel promotes "sense of humor" as one of its key suggestions for teaching middle school mathematics. According to the Editorial Panel, "Humor can be used to communicate to students that it is okay for them to look at things in a offbeat way. It is a great way to increase rapport between the teacher and students." To capture the spirit of motivation, the final chapter in this sourcebook integrates the various Content Standards via mathematics topics that are recreational in nature. To further engage students, humor is used throughout the book in the titles of many activity lessons, in the body of some activity lessons, and as a Mathematical Humor feature with the Teacher's Notes for each lesson. Students will soon see that, in the real world, math can indeed be humorous and fun.

Acknowledgments

I would like to thank my wife, Bonnie, and my children Ben, Jamie, and Joey for all their love, support, and encouragement throughout my career—and in particular, throughout the development of this book. I would also like to thank my sister, Anne Hollenbeck, for creating some of the illustrations used in the book. Finally, I extend a thanks to my editors, Bobbie Dempsey of Good Year Books and my daughter Jamie.

About the Author

David B. Spangler has devoted his entire professional career of more than 35 years to mathematics education. He began as a middle-school mathematics teacher. Later he taught at the community college and university levels, worked as a textbook editor for major publishers, and co-founded a mathematics professional development organization, *Active*Math® Workshops (www.activemath.com). His goal has always been to explore ways to teach mathematics through engaging, real-world applications. He has used many of the activity lessons in this sourcebook with his middle-school students and with teachers in his methods courses and teacher workshops. David's first Good Year Book, *Math for Real Kids,* was recently released in its second edition.

Contents

The Date Detective*

Teacher's Notes

- **NCTM Standards:** Number and Operations
- **CCS Standards:** Operations and Algebraic Thinking; Reason abstractly and quantitatively.
- **Mathematical Topics:** Number Theory (multiples, factors, prime numbers, even and odd numbers, reciprocals, pi, palindromes, perfect numbers)
- **Grouping of Students:** Work independently or in pairs

Background

It's not that I'm so smart, it's just that I stay with problems longer.

—Albert Einstein,
German-born American physicist (1879–1955)

This activity lesson provides an informal way for students to revisit a variety of number theory concepts from the middle-school curriculum. This activity lesson may be used in total, or you may use parts of it as a daily warm-up.

Connecting mathematics concepts to the dates on a calendar reinforces the notion of the "teachable moment." For example, suppose students are learning the concept of "common factors." Suppose also that today's date is, say, 9/30. We could say that today is a *common-factor* date because the month and day numbers have the common factor, 3. You might then ask them to find all common-factor dates during the month of September (or throughout the entire year). Students would thus be reinforcing the day's lesson in a fun, inquisitive way.

Once you permit students to become a "Date Detective," there is no limit to what they are likely to discover! And, they just might enter the classroom each day thinking about mathematics before you even begin your lesson.

For more information on what is special about each of the numbers 1–31, go to http://richardphillips.org.uk/number.

*A source for some of the ideas used in this activity lesson is Larry N. Campbell, "Calendar Search: Every Day Is Special!," edited by David B. Spangler, *Mathematics Teaching in the Middle School* (September 1998), pp. 37–39.

Mathematical Humor

Teacher: There is a difference between having an odd number of students in the room and having a number of odd students in the room. Of course, it's also possible to have an even number of odd students in the room.

★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★

Johnny: Grandma, can you help me find the least common multiple?

Grandmother: Are they still looking for that? They were looking for that when I was your age!

Solutions

1. 1/1; 2/1, 2/2; 3/1, 3/3; 4/1, 4/2, 4/4; 5/1, 5/5; 6/1, 6/2, 6/3, 6/6; 7/1, 7/7; 8/1, 8/2, 8/4, 8/8; 9/1, 9/3, 9/9; 10/1, 10/2, 10/5, 10/10; 11/1, 11/11; 12/1, 12/2, 12/3, 12/4, 12/6, 12/12
2. May: 5/5, 5/10, 5/15, 5/20, 5/25, 5/30; June: 6/6, 6/12, 6/18, 6/24, 6/30; July: 7/7, 7/14, 7/21, 7/28; August: 8/8, 8/16, 8/24
3. 2/2, 2/3, 2/5, 2/7, 2/11, 2/13, 2/17, 2/19, 2/23, 2/29; 3/2. 3/3, 3/5, 3/7, 3/11, 3/13, 3/17, 3/19, 3/23, 3/29, 3/31; 5/2, 5/3, 5/5, 5/7, 5/11, 5/13, 5/17, 5/19, 5/23, 5/29, 5/31; 7/2, 7/3, 7/5, 7/7, 7/11, 7/13, 7/17, 7/19, 7/23, 7/29, 7/31; 11/2, 11/3, 11/5, 11/7, 11/11, 11/13, 11/17, 11/19, 11/23, 11/29
4. The remaining *square-me* dates are 1/1, 2/4, and 3/9, 4/16.

5a. 3/14 (As an interesting sidelight, March 14 is the birthday of Albert Einstein, born in 1879.)

5b. 3/14/15 at 9:26 A.M. or P.M. (since $\pi = 3.1415926\ldots$)
Note: If a rounded value of π is used, the time would be 9:27 (because $\pi = 3.14159265\ldots$, or 3.1415927, to the nearest hundred-millionth).

5c. March 14, 1592 (or perhaps March 14, 1593, if the date is not truncated).

6. 9/9/99 (The next one will be on 1/1/11.)
7. Possible answers include any ten of these pairs: 3/1 & 1/3, 2/6, 3/9, 4/12, 5/15, 6/18, 7/21, 8/24, 9/27, 10/30; 3/2 & 2/3, 4/6, 6/9, 8/12, 10/15, 12/18; 3/3 & 1/1, 2/2, 4/4, 5/5, 6/6, 7/7, 8/8, 9/9, 10/10, 11/11, 12/12; 3/4 & 4/3, 8/6, 12/9; 3/5 & 5/3, 10/6; 3/6 & 2/1, 4/2, 6/3, 8/4, 10/5, 12/6; 3/7 & 7/3; 3/8 & 8/3; 3/9 & 3/1, 6/2, 9/3, 12/4; 3/10 & 10/3; 3/11 & 11/3; 3/12 & 4/1, 8/2, 12/3; 3/15 & 5/1, 10/2; 3/18 & 6/1, 12/2; 3/21 & 7/1; 3/24 & 8/1; 3/27 & 9/1; 3/30 & 1/10

8a. 10/02/2001

8b. Previous palindromic date: 08/31/1380
If leading zeros are NOT used in months/days, but 2-digit years ARE used, there are at least six more palindromic dates: 10/11/01, 10/22/01, 11/11/11, 11/22/11, 12/11/21, 12/22/21.

9. Sample answer: An *odd-number* date can only occur during an odd-numbered month (Jan., March, May, July, Sept., and Nov.). The four 31-day months in that group each have 16 *odd-number* dates. The other two odd-numbered months each have 15 *odd-number* dates. So each year there are 64 + 30, or 94, *odd-number* dates.

Name _____ Date _____

The Date Detective

Take a moment to think about today's date. Is there anything *mathematically special* about it? For example, if today's date were 3/5/08, you could say that this is "sum date"—because the sum of the month and day numbers is equal to the year number. If today's date were 12/4/08, we could say that "today makes a difference"—because the *difference* between the month and the day numbers is equal to the year number.

In this activity lesson, as a "Date Detective," you will apply number theory concepts to explore and uncover mathematical relationships on the calendar. Unless otherwise specified, use only the last two digits for the *years.* Also, unless specified, do not use leading zeros in front of single-digit months/days. OK, let's start. There are dates waiting to be discovered.