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MATHEMATICS EXPLORATIONS

David B. Spangler

STUDENT WORKBOOK

Student Name: _____

Classroom: _____

Teacher: _____

Note: The separate Teacher's Edition includes solutions to all questions, mathematics background for each activity, extension, mathematical humor, and more.

GOOD YEAR BOOKS

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TO THE STUDENT

Little League Coach: You kids are not showing enough effort. I expect each of you to give me 150%.

Player: I hate to say it, coach, but my math teacher told me you can't have more than 100%.

Little League Coach: OK smarty pants, how would you like to sit on the bench 100% of the next game?

Player: Fine with me . . . as long as you play me the other 50% of the game!

You are about to enter the exciting world of mathematics exploration and discovery. We are confident that you will enjoy this adventure—and find it quite worthwhile. The author, a former middle-school math teacher, has devoted his entire 35-year professional career to creating ways to reach students through engaging, real-world mathematics activities.

Each activity in this book calls on you “to put on a detective hat” to do various types of investigative work. You will search for patterns to break codes, discover formulas, conduct math investigations, solve math mysteries, uncover and correct errors and blunders from the media, analyze why math tricks work, use clues to solve problems, and more. While these activities are designed to arouse your curiosity in a fun way, they will also improve your analytical skills—the kinds you need for success in school and later in the world of work.

To capture the spirit of motivation, humor is used throughout the book in the titles of activities and in many of the activities themselves. The Teacher's Edition includes humor on a regular basis. Ask your teacher to share some of that with you. The final chapter presents topics that are recreational in nature to further enhance your mathematical enjoyment.

There is another important goal of this book. In order to be successful in mathematics, you need to be actively engaged—and give 100% effort. This is as true for learning math as it is for singing, dancing, playing a musical instrument, or pitching a baseball. Simply put, you cannot become good at math by watching your teacher or other students do it. So, we hope this book will spark your desire to be an *active* participant in the discovery and exploration of mathematical ideas.

Your voyage is about to begin. Your “carry-on luggage” should include paper, pencil, and a calculator. We wish you much success.

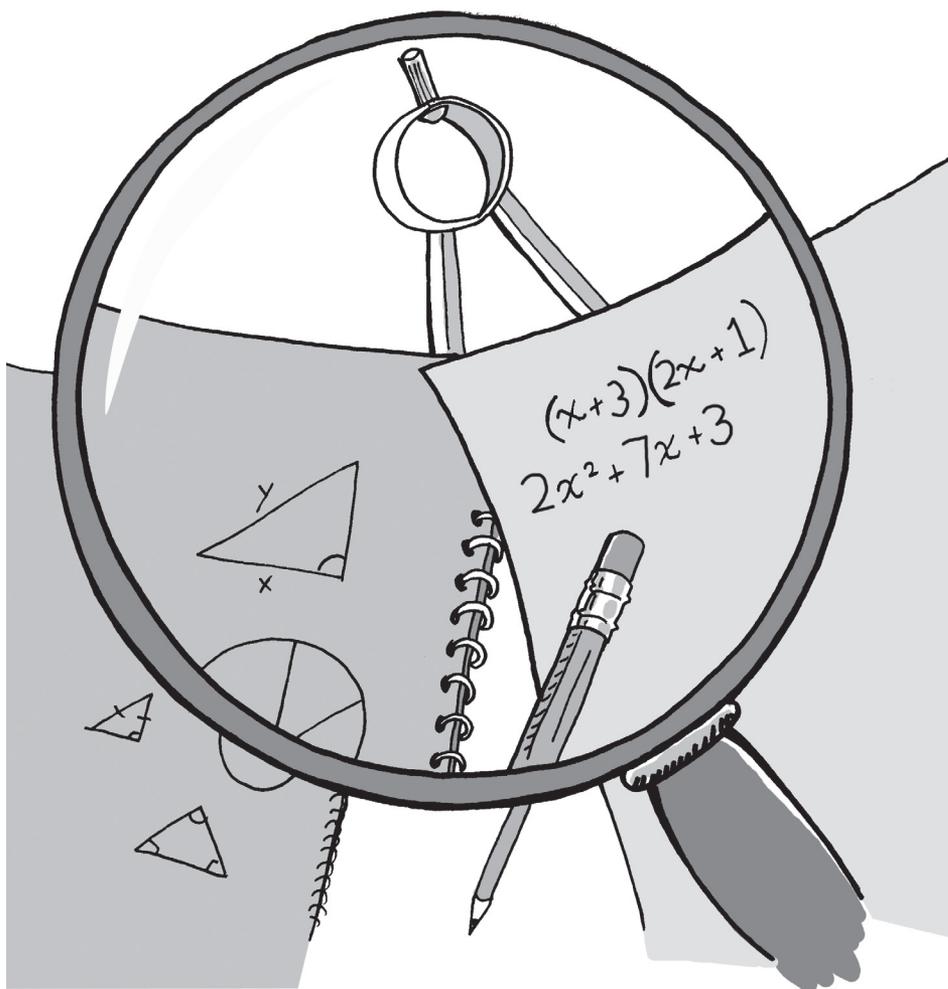
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I would like to thank my wife Bonnie and my children, Ben, Jamie, and Joey, for all their love, support, and encouragement throughout my career—and in particular, throughout the development of this book. I would also like to thank my sister, Anne Hollenbeck, for creating some of the illustrations used in the book. Finally, I extend a thanks to my editors, Bobbie Dempsey of Good Year Books and my daughter Jamie.

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FOCUS ON

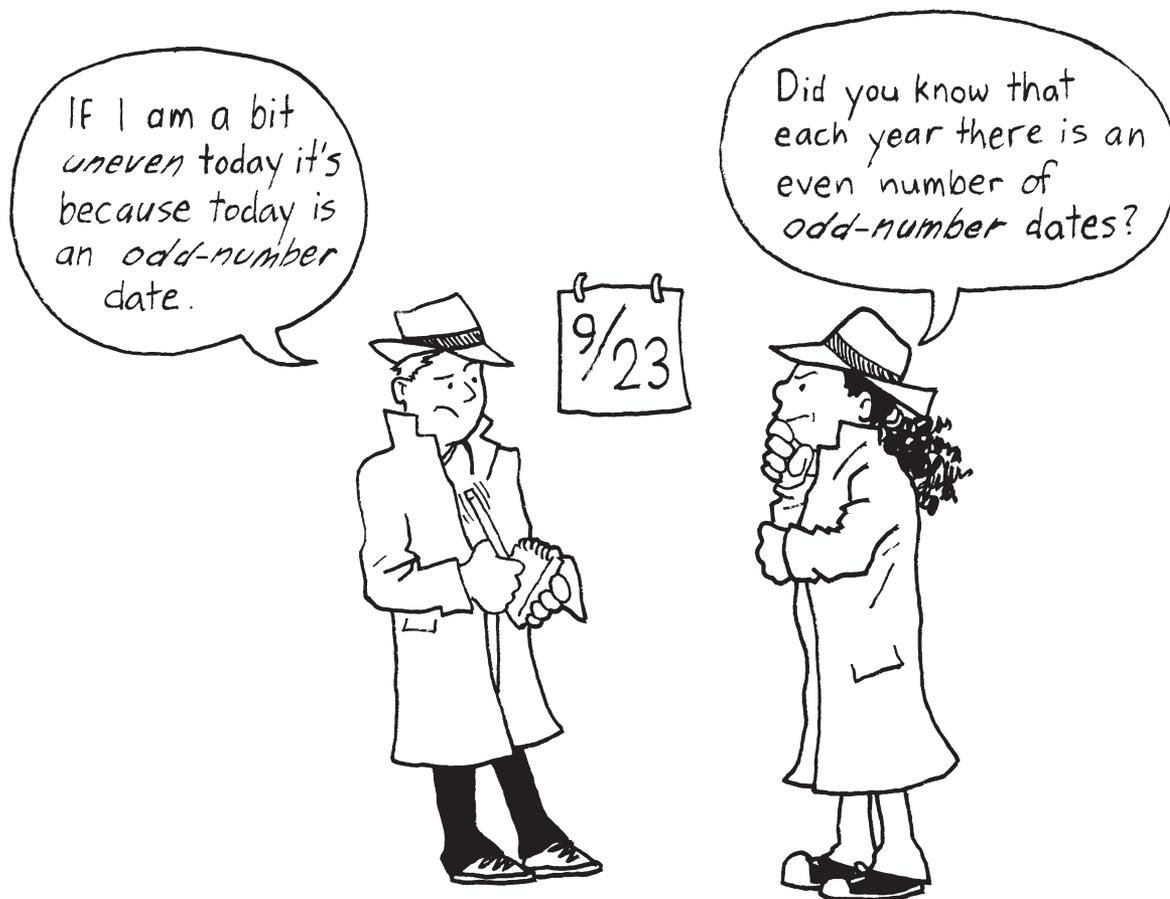
NUMBER AND OPERATIONS



THE DATE DETECTIVE

Take a moment to think about today's date. Is there anything *mathematically special* about it? For example, if today's date were 3/5/08, you could say that this is "sum date"—because the sum of the month and day numbers is equal to the year number. If today's date were 12/4/08, we could say that "today makes a difference"—because the *difference* between the month and the day numbers is equal to the year number.

In this activity lesson, as a "Date Detective," you will apply number theory concepts to explore and uncover mathematical relationships on the calendar. Unless otherwise specified, use only the last two digits for the *years*. Also, unless specified, do not use leading zeros in front of single-digit months/days. OK, let's start. There are dates waiting to be discovered.



QUESTIONS

1. Which dates (month/day) are *multiple* dates? Multiple dates are dates where the month number is a multiple of the day number. For example, 9/3 is a multiple date because 9 is a multiple of 3. Another multiple date is 7/1.

2. A *factor* date is one in which the month number is a *factor* of the day number. For example, 9/18 is a *factor* date because 9 is a factor of 18. Another factor date is 1/7. In fact, all dates in January are *factor* dates because 1 is a factor of all numbers. Which dates (month/day) during May, June, July, and August are *factor* dates?

3. A *prime number* is a whole number greater than 1 that has only itself and 1 as factors. The first five prime numbers are 2, 3, 5, 7, and 11. Which dates (month/day) are *prime* dates? An example of a prime date is 2/3 because both the month and day numbers are prime numbers.

4. There are five *square-me* dates. One of them is 5/25, because the *square* of the month number is the day number. Name the other four square-me dates.

5. *Pi* (π) is the ratio of the circumference to the diameter of any circle. As a decimal, it never ends. $\pi = 3.141592653 \dots$

a. Which date each year (month/day) is *pi* day? _____

b. Which date and time (month/day/year/time) is *pi* time? _____

c. Which date (month/day/4-digit year) was the ultimate *pi* date? _____

6. When was the last date (month/day/year) *in which all digits were the same*? _____

7. Two numbers are *reciprocals* if their product is 1. The fractions (dates) $3/7$ and $7/3$ are reciprocals. Note that pairs of fractions (dates) such as $2/4$ and $12/6$ are also reciprocals (because $\frac{2}{4} \times \frac{12}{6} = 1$).

List ten of the pairs of *reciprocal* dates that occur during the month of March.

8. A number that reads the same both forward and backward is a *palindrome*.

For this problem, use leading zeros in single-digit months/days. Also, use all four digits for the year:

- a. When was the first *palindromic* date (month/day/year) during the twenty-first century? (The twenty-first century began on 01/01/2001.) _____
- b. When was the last *palindromic* date previous to that one? _____
(Hint: It occurred during the fourteenth century.)
9. Review the cartoon at the beginning of this activity lesson. Then verify that there is indeed an even number of *odd-number* dates each year. (A date such as 9/23 is an *odd-number* date because both 9 and 23 are odd numbers.)

10. A *perfect number* is a whole number n for which the sum of all its factors, excluding n , is equal to n . An example of a perfect number is 6. Its factors, less than 6, are 1, 2, and 3. Since $1 + 2 + 3 = 6$, the number 6 is perfect.

Which two dates (month/day) are the only totally *perfect* dates? _____

11. Make up your own types of mathematically special dates.

EDITOR FOR A DAY ERROR SEARCH

This page of math problems contains many errors. Pretend you are a math editor. Your job is to find and correct the errors. In addition to math errors, there are errors in spelling, grammar, punctuation and style. Have you already found some errors?

The answers to the problems are given in bold type. Good luck in your error search.



MULTIPLY OR DIVIDE. SIMPLIFY YOUR ANSWERS.

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. $\frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$

3. $\frac{3}{4} \times \frac{1}{5} = \frac{4}{15}$

4. $4 \times \frac{2}{7} = \frac{8}{28}$

5. $\frac{2}{3} \div \frac{2}{3} = 1$

6. $5 \times \frac{3}{4} = \frac{23}{4}$

7. $\frac{2}{3} \div \frac{3}{4} = \frac{1}{2}$

8. $\frac{1}{8} \div 8 = \frac{1}{64}$

9. $5 \div \frac{5}{6} = \frac{1}{6}$

10. $\frac{9}{10} \times \frac{5}{6} = \frac{3}{4}$

11. $\frac{3}{4} \div \frac{1}{8} = \frac{1}{6}$

12. $0 \times \frac{3}{5} = 0$

14. $\frac{4}{3} \times 4 = 5\frac{1}{3}$

15. $\frac{3}{10} \times \frac{9}{2} = \frac{3}{5}$

16. $\frac{3}{10} \div 7 = 2\frac{1}{10}$

17. $9 \div 18 = \frac{1}{2}$

SOLVE EACH PROBLEM

17. The length of a track around a football field is $\frac{1}{4}$ miles. You jog 6 times around the track. How far do you jog?

$1\frac{1}{2}$

18. Arbor School invited boys to try out for its basketball team. Of the 36 boys who tried out, $\frac{1}{3}$ made the team. How many boys did not make their team?

12 boys

19. Doug ate $\frac{3}{4}$ of a pie. Then he ate $\frac{1}{4}$ of what was left. How much of the pie did she eat?

$\frac{15}{16}$ of the pie

20. Miss Smith's class is cutting ribbon. How many $\frac{1}{2}$ -inch strips can be cut from 20 inches of ribbon.

10 strips